# 1 SEM TDC GEMT (CBCS) GE 1 (A/B/C)

## 2022

( Nov/Dec )

## MATHEMATICS

( Generic Elective )

Paper : GE–1

*The figures in the margin indicate full marks for the questions*

Paper : GE–1 (A)

**( Differential Calculus )**

*Full Marks* : 80
*Pass Marks* : 32

*Time* : 3 hours

1. *(a)* কেতিয়া এটা ফলন $f$ বন্ধ অন্তৰ $[a, b]$ত অনৱচ্ছিন্ন হোৱা বুলি কোৱা হয়? 1

When is a function $f$ said to be continuous in a closed interval $[a, b]$?

*(b)* তলৰ যি কোনো এটাৰ মান নিৰ্ণয় কৰা : 3

Evaluate any *one* of the following :

*(i)* $\lim_{x \to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$

*(ii)* $\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$

(c) $f$ ফলনৰ সংজ্ঞা এনেদৰে দিয়া আছে

$$f(x) = (1+3x)^{1/x}, \quad x \neq 0$$
$$= e^3 \quad , \quad x = 0$$

দেখুওৱা যে $x=0$ বিন্দুত ফলন অনৱচ্ছিন্ন। 3

Show that the function $f$ defined by

$$f(x) = (1+3x)^{1/x}, \quad x \neq 0$$
$$= e^3 \quad , \quad x = 0$$

is continuous at $x=0$.

(d) $y=(ax+b)^m$ৰ $n$-তম অৱকলজ নিৰ্ণয় কৰা য'ত $n \leq m$ আৰু $m, n \in N$. 1

Find the $n$-th derivative of $y=(ax+b)^m$, where $n \leq m$ and $m, n \in N$.

(e) যদি (If)

$$y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$$

দেখুওৱা যে (show that)

$$(1-x^2)y_{n+2} - (2n+3)xy_{n+1} - (n+1)^2 y_n = 0$$ 4

2. লিবনিটজৰ উপপাদ্যটো উল্লেখ কৰা আৰু প্ৰমাণ কৰা। 5

State and prove Leibnitz's theorem.

অথবা / Or

যদি (If)

$$u = \tan^{-1} \frac{x^3 + y^3}{x - y}$$

তেন্তে প্ৰমাণ কৰা যে (then prove that)

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

3. (a) যদি $u = f(xyz)$ হয়, তেন্তে $\frac{\partial f}{\partial y}$ নিৰ্ণয় কৰা। 1

If $u = f(xyz)$, then find $\frac{\partial f}{\partial y}$.

(b) যদি (If)

$$u = \sin^{-1}\left\{\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}\right\}$$

তেন্তে প্ৰমাণ কৰা যে (then prove that)

$$\frac{\partial u}{\partial x} = -\frac{y}{x}\frac{\partial u}{\partial y}$$

4

(c) যদি $y = \sin^2 x$, তেন্তে $y_n$ নিৰ্ণয় কৰা। 1

If $y = \sin^2 x$, then find $y_n$.

4. (a) যদি $f = \tan^{-1}\frac{y}{x}$ হয়, তেন্তে $\frac{\partial f}{\partial x}$ নিৰ্ণয় কৰা। 1

If $f = \tan^{-1}\frac{y}{x}$, then find $\frac{\partial f}{\partial x}$.

(b) দেখুওৱা যে এটা ফলন $f(x) = |x| + |x-1|$, এটা বিন্দু $x = 1$ ত অনৱচ্ছিন্ন কিন্তু অৱকলনীয় নহয়। 3

Show that the function $f$ defined as follows, is continuous but not derivable at $x = 1$, $f(x) = |x| + |x-1|$.

(c) যদি (If)

$$u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

তেন্তে দেখুওৱা যে (then show that)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$ 3

5. (a) $y = x^2(a-x)$ বক্ৰৰ উপস্পৰ্শকৰ দৈৰ্ঘ্য নিৰ্ণয় কৰা। 1

Find the length of the subtangent to the curve $y = x^2(a-x)$.

(b) দেখুওৱা যে, যি কোনো বক্ৰৰ ক্ষেত্ৰত

$$\frac{\text{উপ-অভিলম্ব}}{\text{উপ-স্পৰ্শক}} = \left(\frac{\text{অভিলম্বৰ দীঘ}}{\text{স্পৰ্শকৰ দীঘ}}\right)^2$$ 2

Show that in any curve

$$\frac{\text{subnormal}}{\text{subtangent}} = \left(\frac{\text{length of normal}}{\text{length of tangent}}\right)^2$$

6. (a) যি কোনো বক্ৰৰ ক্ষেত্ৰত উপস্পৰ্শকৰ সংজ্ঞা লিখা। 1

Define subtangent to any curve.

(b) $x = a(\theta + \sin\theta)$ আৰু $y = a(1-\cos\theta)$ বক্ৰৰ $\theta$ত উপস্পৰ্শকৰ দৈৰ্ঘ্য নিৰ্ণয় কৰা। 3

Find the lengths of subtangent to $x = a(\theta + \sin\theta)$ and $y = a(1-\cos\theta)$ at $\theta$.

7. তলত দিয়া বক্ৰৰ অনন্তস্পৰ্শী নিৰ্ণয় কৰা : 4

Find the asymptotes of the following curve :

$$x^3 - 2x^2y + xy^2 + x^2 - xy + 2 = 0$$

অথবা / *Or*

$a^4y^2 = x^4(2x^2 - 3a^2)$ বক্ৰৰ অৱস্থান আৰু দ্বি-বিন্দুৰ প্ৰকৃতি নিৰ্ণয় কৰা।

Find the position and nature of the double points of the curve $a^4y^2 = x^4(2x^2 - 3a^2)$.

8. তলৰ যি কোনো এটাৰ মান নিৰ্ণয় কৰা : 4

Evaluate any *one* of the following :

(a) $y = x(x^2 - 1)$ বক্ৰৰ অনুৰেখন নিৰ্ণয় কৰা।

Trace the curve $y = x(x^2 - 1)$.

(b) দেখুওৱা যে $r = a(1-\cos\theta)$ কাৰডিয়ইড়ৰ যি কোনো বিন্দু $(r, \theta)$ত বক্ৰতা ব্যাসাৰ্ধ $\frac{2}{3}\sqrt{2ar}$.

Show that the radius of curvature at any point $(r, \theta)$ of the cardioid $r = a(1-\cos\theta)$ is given by $\frac{2}{3}\sqrt{2ar}$.

9. $f(x, y) = 0$ বক্ৰৰ যি কোনো বিন্দু $(x, y)$ ত বহু বিন্দু হোৱাৰ প্ৰয়োজনীয় আৰু পৰ্যাপ্ত চৰ্ত উল্লেখ কৰি প্ৰমাণ কৰা। 5

State and prove the necessary and sufficient condition for any point $(x, y)$ on the curve $f(x, y) = 0$ to be a multiple point.

অথবা / Or

এটা বক্ৰৰ কাৰ্টেচিয়ান সমীকৰণ $y = f(x)$ হ'লে বক্ৰৰ এটা বিন্দুত বক্ৰতা ব্যাসাৰ্ধ নিৰ্ণয় কৰা।

Find the radius of curvature at a point of the Cartesian equation of the curve $y = f(x)$.

10. (a) ৰোলৰ উপপাদ্যটো লিখা। 1

State the Rolle's theorem.

(b) $[-1, 1]$ অন্তৰালত $f(x) = \dfrac{1}{2-x^2}$ ফলনৰ বাবে ৰোলৰ উপপাদ্য প্ৰতিপন্ন কৰা। 2

Verify Rolle's theorem for the function

$$f(x) = \frac{1}{2-x^2}$$

in the interval $[-1, 1]$.

(c) মধ্যমান উপপাদ্য $f(b) - f(a) = (b-a)f'(\xi)$ প্ৰতিপন্ন কৰা য'ত $f(x) = x(x-1)(x-3)$, $a = 0$, $b = \dfrac{1}{2}$ আৰু $\xi$ ৰ মান নিৰ্ণয় কৰা। 4

Verify the applicability of the mean value theorem $f(b) - f(a) = (b-a)f'(\xi)$, $a < \xi < b$ if $f(x) = x(x-1)(x-3)$, where $a = 0$, $b = \dfrac{1}{2}$. Also find the value of $\xi$.

**11.** লাগ্ৰাঞ্জৰ মধ্যমান উপপাদ্য উল্লেখ কৰি প্ৰমাণ কৰা। 1+4=5

State and prove Lagrange's mean value theorem.

অথবা / Or

মেক্লৰিনৰ উপপাদ্য ব্যৱহাৰ কৰি $\sin x$ক $x$-ৰ সূচকত অসীম শ্ৰেণীত বিস্তৃতি কৰা। 5

Using Maclaurin's theorem, expand $\sin x$ in an infinite series in powers of $x$.

**12.** (a) যদি (If)

$$f(x) = f(0) + xf'(0) + \frac{x^2}{\underline{|2}} f''(\theta x)$$

তেন্তে θৰ মান উলিওৱা যেতিয়া $x \to 1$ আৰু য'ত $f(x) = (1-x)^{5/2}$.

then find $\theta$ when $x \to 1$ and where $f(x) = (1-x)^{5/2}$. 3

(b) $f(x, y) = x^3 + y^3 - 3x - 12x + 20$ ফলনৰ সৰ্বোচ্চ আৰু সৰ্বনিম্ন মান নিৰ্ণয় কৰা। 4

Find the maximum and minimum values of the function

$$f(x, y) = x^3 + y^3 - 3x - 12x + 20$$

**13.** *(a)* $\log x$ক $x-1$ৰ সূচকত বিস্তৃতি কৰা য'ত $0 < x \leq 2$. 1

Expand $\log x$ in powers of $x-1$ where $0 < x \leq 2$.

*(b)* তলৰ যি কোনো এটাৰ মান নিৰ্ণয় কৰা : 4

Evaluate any *one* of the following :

*(i)* $\lim_{x \to 1} \left\{ \frac{x}{x-1} - \frac{1}{\log x} \right\}$

*(ii)* $\lim_{x \to 0} (\cos x)^{\cot^2 x}$

**14.** *(a)* লাগ্ৰাঞ্জৰ ৰূপৰ অৱশেষ থকা মেক্‌লৰিনৰ উপপাদ্য লিখা। 1

Write the Maclaurin's theorem with Lagrange's form of remainder.

*(b)* মেক্‌লৰিনৰ অসীম শ্ৰেণী ব্যৱহাৰ কৰি $\log(1+x)$ৰ বিস্তৃতি কৰা য'ত $-1 < x < 1$. 5

Expand $\log(1+x)$ using Maclaurin's infinite series where $-1 < x < 1$.

অথবা / *Or*

লাগ্ৰাঞ্জৰ ৰূপৰ অৱশেষ থকা টেইলৰৰ উপপাদ্য লিখি প্ৰমাণ কৰা।

State and prove Taylor's theorem with Lagrange's form of remainder.

Paper : GE–1 (B)

**( Object-Oriented Programming in C++ )**

*Full Marks* : 60

*Pass Marks* : 24

*Time* : 3 hours

**1.** Answer the following questions : 1×5=5

*(a)* Define abstraction.

*(b)* State one difference between C and C++.

*(c)* Write one characteristic of object-oriented programming language.

*(d)* What is the use of <iostream.h>?

*(e)* How are objects created from a class?

**2.** Answer any *five* of the following questions : 2×5=10

*(a)* When do you declare a method or class abstract?

*(b)* Briefly explain the structure of C++ program.

*(c)* How does inheritance help us to create new classes?

*(d)* Why can we not override static method?

*(e)* State the difference between while loop and do while loop.

*(f)* Define default constructor and copy constructor.

**3.** Answer any *five* of the following questions : 3×5=15

*(a)* Explain the following operators and their uses :

cin, cout and delete.

*(b)* Explain the three access modifiers.

*(c)* What is dynamic binding? Define message passing.

*(d)* State the difference between break and continue with example.

*(e)* Define file pointer. What is function prototyping? Explain with example.

*(f)* Explain the increment and decrement operators in brief.

**4.** Answer any *four* of the following questions : 5×4=20

*(a)* Write a C++ program to store information of a book in a structure.

*(b)* Write a C++ program to overload a unary operator.

*(c)* Write a C++ program to display Fibonacci series up to 50.

*(d)* Write a C++ program to implement friend function.

*(e)* Write a C++ program to count the number of objects created.

**5.** *(a)* Explain the different types of inheritance with examples and diagrams. 10

*Or*

*(b)* Explain inline and virtual functions with suitable example.

Paper : GE–1 (C)

**( Finite Element Methods )**

*Full Marks* : 80

*Pass Marks* : 32

*Time* : 3 hours

1. *(a)* Write True or False : 1

The finite-element method is a piecewise application of a variational method.

*(b)* Write down the differences between finite difference methods and finite element methods. 3

*(c)* Consider the boundary value problem

$$u'' + (1 + x^2)u + 1 = 0$$

Determine the coefficients of the approximate solution

$$W(x) = a_1(1 - x^2) + a_2 x^2 (1 - x^2)$$

by using the least square method. 5

*Or*

Using Galerkin's method, solve the boundary value problem

$$\nabla^2 u = -1, \quad |x| \le 1, \quad |y| \le 1$$
$$u = 0, \quad |x| = 1, \quad |y| = 1$$

with $h = \frac{1}{2}$.

Paper : GE–1 (C)

**( Finite Element Methods )**

*Full Marks* : 80
*Pass Marks* : 32

*Time* : 3 hours

1. *(a)* Write True or False : 1
The finite-element method is a piecewise application of a variational method.

*(b)* Write down the differences between finite difference methods and finite element methods. 3

*(c)* Consider the boundary value problem

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$$u = 0, \quad |x| = 1, \quad |y| = 1$$

with $h = \frac{1}{2}$.





*(d)* Find the variational functional for the boundary value problem

$$u'' = u - 4xe^x$$
$$u'(0) - u(0) = 1, \quad u'(1) + u(1) = -e$$

5

*(e)* State and prove the Lax-Milgram theorem. 6

2. *(a)* The application of the finite element method to the boundary value problem

$$-u'' = x$$
$$u(0) = u(1) = 0$$

leads to the system of equations $Au = b$. Determine the matrix $A$ and the column vector $b$ for four elements of equal lengths. 6

*(b)* Apply Galerkin method to the boundary value problem

$$\nabla^2 u + \lambda u = 0, \quad |x| \le 1, \quad |y| \le 1$$
$$u = 0, \quad |x| = 1, \quad |y| = 1$$

to get the characteristic equation in the form $|A - \lambda B| = 0$. 6

3. *(a)* Define assembly of the element equations. 1

*(b)* Define two principles that were used in one-dimensional problem to assembly of finite element equations. 2

(c) Discuss briefly with an example about the element assemblage in finite element method. 3

(d) Write down the importance of sparse matrix in the process of element assemblage with an example. 4

(e) If the finite solutions at any point $(x, y)$ in an element $\Omega^e$ is given by

$$U(x, y) = \sum_{J=1}^{n} U_J^e \psi_J^e (x, y)$$

Find its derivatives. 2

**4.** (a) State the properties for a quadratic triangular element. 3

(b) Give an example of triangular element with a common node. 1

(c) Illustrate the process of discretization in two-dimensional domain with a suitable example. 5

(d) Write the importance of isoperimetric element in the process of element assemblage with an example. 3

**5.** (a) Write True or False : 1

Finite element modelling involves assumptions concerning the representation of the system and its behaviour.

*( Continued )*

*(c)* Discuss briefly with an example about the element assemblage in finite element method. 3

*(d)* Write down the importance of sparse matrix in the process of element assemblage with an example. 4

*(e)* If the finite solutions at any point $(x, y)$ in an element $\Omega^e$ is given by

$$U(x, y) = \sum_{J=1}^{n} U_J^e \psi_J^e (x, y)$$

Find its derivatives. 2

**4.** *(a)* State the properties for a quadratic triangular element. 3

*(b)* Give an example of triangular element with a common node. 1

*(c)* Illustrate the process of discretization in two-dimensional domain with a suitable example. 5

*(d)* Write the importance of isoperimetric element in the process of element assemblage with an example. 3

**5.** *(a)* Write True or False : 1

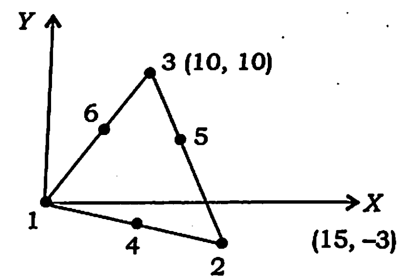
Finite element modelling involves assumptions concerning the representation of the system and its behaviour.





*(b)* Write about interpolating function in finite element method. Find an expression for interpolating function in one-dimensional domain. 3

*(c)* Calculate the interpolation function for the quadratic triangular element shown in the figure : 4



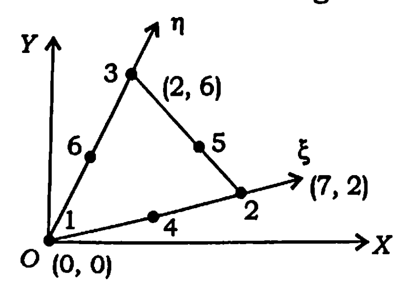

*(d)* Evaluate the integral of the form

$$I = \int_{(e)} F(x)\, dx$$

for the triangular element where $F(x)$ is given function, $(e)$ is the element and $x$ represents multidimensional coordinates. 4

*Or*

Consider the quadratic triangular element shown in the figure :



Evaluate the integral of the product

$$\left(\frac{\partial\psi_1}{\partial x}\right)\left(\frac{\partial\psi_4}{\partial x}\right)$$

at the point $(x, y) = (2, 4)$.

6. (a) What are the different types of partial differential equations? Write their field in applications. 4

(b) Find the solution of the boundary value problem

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{1+e^u}{2}=0, \quad |x|\le 1, \quad |y|\le 1$$

$$u=0, \quad |x|=1, \quad |y|=1$$

by finite element method (use the three node triangular element). 4

(c) Use finite element method to solve the boundary value problem

$$\nabla^2 u=-1, \quad |x|\le 1, \quad |y|\le 1$$

$$\frac{\partial u}{\partial x}+u=0, \quad |x|=1, \quad |y|=1$$

with $h=\frac{1}{2}$. 4

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