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3 SEM TDC GEMT (CBCS) GE 3 (A/B/C)

2021

(Held in January/February, 2022)

MATHEMATICS

(Generic Elective)

Paper : GE–3

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

Paper : GE–3A

(Real Analysis)

1. *(a)* Write an example of countable set. 1

(b) Prove that the set Q of all rational numbers is denumerable. 3

(c) Find all $x \in R$, that satisfy the following inequalities : 2

$$|4x-5| \leq 13$$

(d) Find the supremum and infimum, if they exist, of the following set : 4

$$\left\{\frac{n}{n+1}; n \in N\right\}$$

Or

Let S be a set that is bounded below. Prove that a lower bound w of S is the infimum of S if and only if for any $\varepsilon > 0$ there exists $t \in S$, such that $t < w + \varepsilon$.

2. (a) Write the completeness property of R. 1

(b) If $y > 0$, there exist $n_y \in N$, prove that $n_y - 1 \leq y < n_y$. 2

(c) If S is a subset of R that contains at least two points and has the property $x, y \in S$ and $x < y$, then prove that S is an interval. 3

(d) If M and N are two neighbourhoods of a point x, then show that $M \cap N$ is also neighbourhood of x. 4

Or

If $I_n = [a_n, b_n]$, $n \in N$ is nested sequence of closed bounded intervals, then there exists a number $\xi \in R$; prove that $\xi \in I_n$, $\forall n \in N$.

3. *(a)* Define range of a sequence. 1

(b) Prove that a sequence in R can have at most one limit. 3

(c) Prove that the sequence

$$\{x_n\} = \left\{\frac{3n+1}{n+2}\right\}$$

is monotonic increasing. 2

(d) Establish the convergence or the divergence of the sequence $\{x_n\}$, where

$$x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}, \text{ for } n \in N \quad 4$$

Or

Let $X = \{x_n\}$ and $Y = \{y_n\}$ be sequence of real numbers that converges to x and y respectively. Then prove that the sequences $X+Y$ and XY converges to $x+y$ and xy respectively.

4. *(a)* Define monotonic sequence. 1

(b) Prove that the sequence $\{1+(-1)^n\}$ is divergent. 2

(c) Prove that the sequence

$$\left\{\frac{n+1}{n}\right\}$$

is a Cauchy sequence. 3

(d) State and prove Cauchy's convergence criterion. 4

Or

If $X = \{x_n\}, Y = \{y_n\}$ and $Z = \{z_n\}$ are sequences of real numbers such that $x_n \leq y_n \leq z_n$, for all $n \in N$ and that $\lim x_n = \lim z_n = l$, then prove that $Y = \{y_n\}$ is convergent and $\lim y_n = l$.

5. (a) If the series $\sum_{n=1}^{\infty} x_n$ converges, then write the value of $\lim_{n \to \infty} x_n$. 1

(b) Write an example of an alternating series. 1

(c) Write the condition of root test for convergence of an infinite series. 2

(d) Prove that the series

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

diverges. 3

Or

Prove that the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$$

is convergent.

(e) If a series in R absolutely convergent, then prove that it is convergent. 3

6. Test the convergence of any *two* of the following : 5×2=10

(i) $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$

(ii) $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + 1} x^n, \; x > 0$

(iii) $\frac{1 \cdot 2}{3^2 \cdot 4^2} + \frac{3 \cdot 4}{5^2 \cdot 6^2} + \frac{5 \cdot 6}{7^2 \cdot 8^2} + \cdots \text{ to } \infty$

(iv) $1 + \frac{2^P}{\lfloor 2} + \frac{3^P}{\lfloor 3} + \frac{4^P}{\lfloor 4} + \cdots \text{ to } \infty \; \forall P \in R$

7. *(a)* Define pointwise convergence of sequence of function. 1

(b) Prove that

$$\lim_{n\to\infty} \frac{x^2 + nx}{n} = x$$

for $x \in R$. 2

(c) Show that a sequence $\{f_n\}$ of bounded function on $A \subseteq R$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$. 3

(d) Show that the sequence $\{f_n\}$, where $f_n(x) = x^n$ is uniformly convergent on $[0, k]$; $k < 1$ and only convergent on $[0, 1]$. 4

Or

Let M_n be the sequence of positive real numbers; such that $|f_n(x)| \leq M_n$ for all $n \in N$ and for all $x \in D \subseteq R$. If the series ΣM_n is convergent, then prove that Σf_n is uniformly convergent on D.

8. *(a)* Define radius of convergence of a power series. 1

(b) Determine the radius of convergence of the series $\Sigma a_n x^n$ using any *one* of the following : 4

(i) $a_n = \frac{1}{n^n}$

(ii) $a_n = \frac{m^n}{\lfloor n}$

(c) Show that the series for which

$$f_n(x) = \frac{nx}{\cdot 1 + n^2 x^2}, \quad x \in [0, 1]$$

cannot be differentiated term-by-term at $x = 0$. 5

Or

Let $\{f_n\}$ be a sequence of function in $R[a, b]$ and suppose that $\{f_n\}$ converges uniformly to f. Prove that

$$f \in R[a, b], \text{ i.e., } \int_a^b f = \lim_{n \to \infty} \int_a^b f_n$$

Paper : GE–3B

(Cryptography and Network Security)

1. *(a)* Define cryptography. What are private key cryptography and public key cryptography? Briefly explain with example. 3+3+3=9

   (b) Briefly explain encryption and decryption with suitable example. 3+3=6

   (c) Define hash function and message digest with suitable example. 3+3=6

   (d) What are MAC and HMAC? 3+3=6

2. Illustrate the working principle of digital signature. 10

   Or

   Explain the working mechanisms of RSA.

3. Define any *one* from the following : 5

   (a) IP Spoofing

   (b) MD5

   (c) SYN Flood

   (d) SNMPV1

4. What is ICMP? Briefly explain three different ICMP messages. 2+(2×3)=8

Or

Briefly explain denial of service attack. 8

5. *(a)* What is VPN and why is it needed? 3+3=6

(b) Why does IPSEC need a security association? 4

(c) What is the purpose of a firewall? 4

6. Briefly explain the working principles of (any *two*) : 8×2=16

(a) Packet Filtering Router

(b) Level Gateway or Proxy

(c) Content Filters

(d) Bastion Host

Paper : GE–3C

(Information Security)

1. Answer the following / Fill in the blank : 1×8=8

(a) State one merit of Diffie-Hellman key-exchange algorithm.

(b) What is encryption?

(c) State the term 'buffer overflow'.

(d) List the three classes of intruders.

(e) The encrypted text is also called ____.

(f) How does digital signature differ from authentication protocol?

(g) Define digital signature.

(h) What is Trojan horse?

2. Answer the following : 2×8=16

(a) Why is trap door one-way function used?

(b) State any two modes of operation of block cipher.

(c) Define security threats.

(d) Specify the components of encryption algorithm.

(e) Compare between public key signatures and symmetric key signatures.

(f) Explain how the integrity of message is ensured without source authentication.

(g) What is auditing and lagging in information security?

(h) Draw the block diagram of MD5 message digest algorithm.

3. Define confidentiality, integrity and availability. Explain with diagram. 5+5=10

4. Answer the following :

(a) Explain RSA technique with an example. 6

(b) Describe AES algorithm. 5

Or

Explain briefly about Diffie-Hellman key-exchange algorithm.

(c) Describe cryptographic hash function. 10

(d) State the properties of digital signature. 5

5. Answer any *two* of the following : 10×2=20

(a) Explain the different aspects of security with examples.

(b) Explain the causes of trapdoors.

(c) Analyze various types of attacks of information security.

(d) State the purpose of use of proxy firewall.

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