6 SEM TDC DSE MTH (CBCS) 6 (H)

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(June/July)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–6

(Mathematical Methods)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Write the period of $\sin 2x$. 1

   (b) Write two sufficient conditions for a function to be expanded in a Fourier series. 2

   (c) Find the Fourier series for the function $f(x) = \pi + x$ in the interval $(0,\ 2\pi)$. 7

(Turn Over)

Or

Find the Fourier series for the function $f(x) = x$ in the interval $(0, \pi)$.

2. *(a)* Write the value of $L\{t^2\}$.   1

    (b) Write the value of $L\{e^{4t}\}$.   1

    (c) Find $L\{\sin 2t\}$.   2

    (d) Find any *four* of the following :   4×4=16

        (i) $L\{\cosh^2 2t\}$

        (ii) $L\{e^{3t} \cos 2t\}$

        (iii) $L\{(t^2 + 1)^2\}$

        (iv) $L\{(1 + te^t)^3\}$

        (v) $L\{\sin^3 2t\}$

        (vi) $L\{t\sin^2 2t\}$

3. *(a)* Write the value of $L^{-1}\{2\}$.   1

    (b) Write the value of $L^{-1}\left\{\dfrac{S}{S^2 + 9}\right\}$.   1

    (c) Find any *two* of the following :   4×2=8

        (i) $L^{-1}\left\{\dfrac{1}{(S+1)(S+2)}\right\}$

        (ii) $L^{-1}\left\{\dfrac{S^2}{(S+2)^3}\right\}$

(iii) $L^{-1}\left\{\dfrac{S}{(S+1)^{\frac{3}{2}}}\right\}$

(iv) $L^{-1}\left\{\log\dfrac{S+6}{S+2}\right\}$

4. *(a)* Write the Fourier sine integral formula.   1

(b) Write the Dirichlet's conditions of Fourier transform.   2

(c) Answer any *three* of the following : 5×3=15

   (i) State and prove the change of scale property of Fourier transform.

   (ii) Find the Fourier transform of

   $$f(x) = e^{-\frac{x^2}{2}}, \quad -\infty < x < \infty$$

   (iii) Find the inverse Fourier transform of $e^{-|s|\,y}$.

   (iv) Show that

   $$F\{x^n f(x)\} = (-i)^n \frac{d^n}{ds^n}(\bar{f}(s))$$

(d) Find the Fourier sine and cosine transforms of $f(x) = x$.   7

<center>Or</center>

Find the Fourier transform of

$$f(x) = \begin{cases} \cos x, & \text{for} \quad |x| < a \\ 0, & \text{for} \quad |x| > a > 0 \end{cases}$$

5. *(a)* Write the value of $L\left\{\dfrac{\partial y}{\partial t}\right\}$.   1

     (b) Find $L\left\{\dfrac{\partial y}{\partial x}\right\}$.   2

     (c) Solve any *two* of the following, using Laplace transform :   6×2=12

       (i) $\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} + 5y = 5,\ y(0) = y'(0) = 0$

       (ii) $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} - 3y = \sin t,$

                   $y(0) = y'(0) = 0$

       (iii) $\dfrac{d^2y}{dt^2} + y = 0,\ y(0) = 1,\ y'(0) = 0$

       (iv) $\dfrac{dx}{dt} + y = \sin t,\ \dfrac{dy}{dt} + x = \cos t$

                  $x(0) = 2,\ y(0) = 2$

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