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6 SEM TDC DSE MTH (CBCS) 1 (H)

2022

(June/July)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–1

(Hydromechanics)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Define real fluid. 1

(b) Fill in the blank : 1

The motion of a fluid is said to be ____ when the vorticity of every fluid particle is zero.

(c) Write the difference between stream-lines and path lines. 3

(d) Determine the acceleration at the point (2, 1, 3) at $t = 0 \cdot 5$, if $u = yz + t$, $v = xz - t$ and $w = xy$. 3

(e) Deduce the equation of continuity in Cartesian coordinates. 7

Or

Show that a fluid of constant density can have a velocity with components

$$u = -\frac{2xyz}{(x^2 + y^2)^2}, \quad v = \frac{(x^2 - y^2)z}{(x^2 + y^2)^2},$$

$$w = \frac{y}{x^2 + y^2}$$

Find the vorticity vector.

2. *(a)* What is called conservative field of force? 1

(b) Write down the Bernoulli's equation for steady and irrotational flow. 1

(c) State and prove Kelvin's circulation theorem. 5

(d) A sphere of radius a is surrounded by infinite liquid of density ρ, the pressure at infinity being Π. Show that the pressure at a distance r from the centre immediately falls to $\Pi(1 - a/r)$. 5

Or

Deduce the equation of motion for impulsive force.

3. *(a)* Define cyclic irrotational motion. 1

(b) State whether True or False : 1

Acyclic irrotational motion is possible in a liquid bounded entirely by fixed rigid walls.

(c) State and prove Kelvin's minimum energy theorem. 6

Or

A velocity field is given by $\vec{q} = \frac{(-y\hat{i} + x\hat{j})}{x^2 + y^2}$.

Calculate the circulation round a square with its corners at (1, 0), (2, 0), (2, 1) and (1, 1).

4. *(a)* Define specific gravity and density of a fluid. 2

(b) Write True or False : 1

In a fluid, at rest under gravity, the pressure is the same at all points in the same horizontal plane.

(c) Prove that surfaces of equal pressure are intersected orthogonally by the lines of forces. 3

(d) The liquids A and B do not mix and have different densities. When A is poured in a vessel to vertical height h, the pressure on the bottom is the same as when A stands to a height h_1 and B to height h_2 above it. Show that the ratio of the densities of A to that of B is $h_2/(h-h_1)$. 5

(e) A tube in the form of a parabola held its vertex downwards and axis vertical is filled with two different liquids of densities ρ and ρ'. If the distances of the free surfaces of the liquids from the focus are r and r' respectively, show that the distance of the common surface from the focus is $\dfrac{r\rho - r'\rho'}{\rho - \rho'}$. 6

Or

State and prove the necessary and sufficient condition that a given distribution of forces (X, Y, Z) can keep a liquid in equilibrium.

5. (a) Define whole pressure. 2

(b) Describe centre of pressure. 1

(c) Find the CP of a parallelogram immersed in a homogeneous liquid with one side in the free surface. 6

(d) A hemispherical bowl is filled with water and inverted, and placed with its plane base in contact with a horizontal table. Find the resultant thrust on its surface. Also show that the resultant vertical thrust on its surface is one-third of the thrust on the table. 7

Or

A hollow cone is placed with its vertex upward on a horizontal table and liquid is poured in through a small hole in the vertex. If the cone begins to rise when the weight of the liquid poured in it equals its weight, prove that its weight is to the weight of the liquid required to fill the cone is as $(9-3\sqrt{3}):4$.

6. (a) State the condition of equilibrium of a floating body. 2

(b) Fill in the blank : 1

If the _____ coincide with the centre of gravity then equilibrium is neutral.

(c) Define metacentre. 1

(d) Write the forces acting on a body immersed in a liquid and supported by a string. 3

(e) A solid cone of semivertical angle α, specific gravity σ floats in equilibrium in the liquid of specific gravity ρ with its axis vertical and vertex downwards. Determine the condition for which the equilibrium is stable. 5

Or

Prove that the tangent at any point of surface of buoyancy is parallel to the corresponding plane of floatation.

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